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ON THE THREE-DIMENSIONAL PROBLEM OF MAGNETOELASTIC PLATE VIBRATIONS

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The problem of investigating the magnetoelastic vibrations of an electrically conducting plate in a magnetic field reduces to the combined solution of the magnetoelasticity equations in the domain occupied by the plate (interior problem), and the electrodynamics equations of the rest of the domain of the space under consideration (exterior problem).

An attempt is made to determine the magnetic field of a thin plate of finite conductivity by the asymptotic integration of the combined equations of magnetoelasticity for the domain occupied by the plate. Jointly considering the exterior and interior problems, the magnetoelastic vibrations of a thin plate of finite conductivity are investigated. Some magnetoelasticity hypotheses are formulated for a plate of finite conductivity.

In particular cases when the plate material is ideally conductive or a thin plate of infinite extent has finite electrical conductivity, the problem of the magnetoelastic vibrations is solved relatively simply [1. 2].

In the general case when the plate can have finite dimensions, and its material is finitely conductive, the solution of the problem posed becomes quite difficult, because the interior problem in this case does not separate, and the exact determination of the magnetic field of the plate in a three-dimensional formulation is not possible.

1. An isotropic elastic plate of constant thickness $2h$ fabricated from a material with finite electrical conductivity and in an external magnetic field with given intensity vector $\mathbf{H}_0$ (H_1, H_2, H_3) is considered.

It is assumed that the magnetic and dielectric permeability of the plate equal one.

The Maxwell equations for a vacuum [3] are considered valid for the exterior domain (for the whole domain outside the plate).

Let us introduce a Cartesian coordinate system (x, y, z) such that the middle plane of the plate would coincide with the xy coordinate plane. In the coordinate system selected, the three-dimensional problem of magnetoelasticity reduces to the combined integration of the following systems of differential equations [3, 4].

In the interior domain:

Electrodynamic equations

$$\operatorname{rot}\mathbf{H} = \frac{4\pi\sigma}{c}\left[\mathbf{E} + \frac{1}{c}\frac{\partial\mathbf{U}}{\partial t}\times\mathbf{H}\right] + \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t} \tag{1.1}$$

$$\operatorname{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t}, \qquad \operatorname{div}\mathbf{H} = 0, \qquad \operatorname{div}\mathbf{E} = 0$$

Equilibrium equations of elasticity theory

$$\frac{\partial\sigma_x}{\partial x} + \frac{\partial\tau_{xy}}{\partial y} + \frac{\partial\tau_{xz}}{\partial z} + X = 0 \qquad (xy)$$

$$\frac{\partial\tau_{xz}}{\partial x} + \frac{\partial\tau_{yz}}{\partial y} + \frac{\partial\sigma_z}{\partial z} + Z = 0 \tag{1.2}$$

$$\mathbf{R}(X, Y, Z) = \frac{\sigma}{c}\left[\mathbf{E} + \frac{1}{c}\frac{\partial\mathbf{U}}{\partial t}\times\mathbf{H}\right]\times\mathbf{H} - \rho\frac{\partial^2\mathbf{U}}{\partial t^2}$$

In the exterior domain:

Electrodynamics equations

$$\operatorname{rot}\mathbf{H}^{(n)} = \frac{1}{c}\frac{\partial\mathbf{E}^{(n)}}{\partial t}, \qquad \operatorname{rot}\mathbf{E}^{(n)} = -\frac{1}{c}\frac{\partial\mathbf{H}^{(n)}}{\partial t} \tag{1.3}$$

(for $z \geqslant h$ the superscript is $n = 1$, and for $z \leqslant -h$ the superscript is $n = 2$).

In the equations presented above $\mathbf{H}$ and $\mathbf{E}$ are, respectively, the magnetic and electrical field intensity vectors, $\mathbf{U} = \mathbf{U}(u_x, u_y, u_z)$ is the displacement vector of the plate particles, $\sigma = \sigma(x, y, t)$ is the conductivity of the plate material, c the velocity of light in vacuum $\sigma_x = \sigma_x(x, y, z, t), \ldots, \tau_{xz} = \tau_{xz}(x, y, z, t)$ are the stress tensor components, $\mathbf{R}$ the vector of volume forces, ρ the density of the plate material, and t the time.

We shall henceforth limit ourselves to an investigation of the question of the magnetoelastic vibrations under small perturbations. As is known [1–3], in this case (1.1) and (1.2) can be linearized.

Taking

$$\mathbf{H} = \mathbf{H}(H_1 + h_x, H_2 + h_y, H_3 + h_z), \qquad \mathbf{E} = \mathbf{E}(E_x, E_y, E_z)$$

for the perturbed electromagnetic field components, and assuming that the induced electromagnetic field components $h_x, h_y, h_z; E_x, E_y, E_z$ are small, (1.1) and (1.2) can be reduced to the following form

$$\frac{\partial h_z}{\partial y} - \frac{\partial h_y}{\partial z} = \frac{4\pi\sigma}{c}\left[E_x + \frac{1}{c}\left(H_3\frac{\partial u_y}{\partial t} - H_2\frac{\partial u_z}{\partial t}\right)\right] + \frac{1}{c}\frac{\partial E_x}{\partial t}$$

$$\frac{\partial h_x}{\partial z} - \frac{\partial h_z}{\partial x} = \frac{4\pi\sigma}{c}\left[E_y + \frac{1}{c}\left(H_1\frac{\partial u_z}{\partial t} - H_3\frac{\partial u_x}{\partial t}\right)\right] + \frac{1}{c}\frac{\partial E_y}{\partial t} \tag{1.4}$$

$$\frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y} = \frac{4\pi\sigma}{c}\left[E_z + \frac{1}{c}\left(H_2\frac{\partial u_x}{\partial t} - H_1\frac{\partial u_y}{\partial t}\right)\right] + \frac{1}{c}\frac{\partial E_z}{\partial t}$$

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\frac{1}{c}\frac{\partial h_x}{\partial t}, \qquad \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -\frac{1}{c}\frac{\partial h_y}{\partial t}$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\frac{1}{c}\frac{\partial h_z}{\partial t} \qquad \text{(cont.)}$$

$$\frac{\partial h_x}{\partial x} + \frac{\partial h_y}{\partial y} + \frac{\partial h_z}{\partial z} = 0, \qquad \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0$$

as well as

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = \rho\frac{\partial^2 u_x}{\partial t^2} + \frac{\sigma}{c}(H_3^2 + H_2^2)\frac{\partial u_x}{\partial t} - \frac{\sigma}{c}\left[H_3\left(E_y + H_1\frac{\partial u_z}{\partial t}\right) - H_2\left(E_z - H_1\frac{\partial u_y}{\partial t}\right)\right]$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} = \rho\frac{\partial^2 u_y}{\partial t^2} + \frac{\sigma}{c}(H_1^2 + H_3^2)\frac{\partial u_y}{\partial t} - \frac{\sigma}{c}\left[H_1\left(E_z + H_2\frac{\partial u_x}{\partial t}\right) - H_3\left(E_x - H_2\frac{\partial u_z}{\partial t}\right)\right] \qquad (1.5)$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} = \rho\frac{\partial^2 u_z}{\partial t^2} + \frac{\sigma}{c}(H_2^2 + H_1^2)\frac{\partial u_z}{\partial t} - \frac{\sigma}{c}\left[H_2\left(E_x + H_3\frac{\partial u_y}{\partial t}\right) - H_1\left(E_y - H_3\frac{\partial u_x}{\partial t}\right)\right]$$

Therefore, the magnetoelasticity problem has been reduced to the integration of the Eqs. (1.3)–(1.5) taking account of the continuity of the electromagnetic field intensity components, the loading conditions of the exterior planes of the plate, and the boundary conditions.

2. Let us consider the interior problem. Let us apply the method of asymptotic integration of the system (1.4), (1.5) by limiting ourselves here to a construction of just the fundamental iteration process [5]. As is known, the fundamental iteration process affords an opportunity to determine the slowly damped part of the solution, which in the case of the plate bending problem, for example, permits determination of the state of stress which the classical theory of plate bending yields in a first approximation, i.e. reduces to the two-dimensional problem of elasticity theory obtained on the basis of the hypothesis of undeformable normals [5].

Following [5], it is assumed that the magnetic and electrical field intensities do not vary too rapidly in the x and y variables, but should evidently change rapidly in the z variable. Expanding the scale in the z variable according to the formula

$$z = h\zeta \qquad (2.1)$$

it is assumed that the rapidity of variation of the magnetic and electrical field intensities in the (x, y, ζ) variables will not be too great. After substituting (2.1), the linearized equations (1.4) take the form

$$\frac{\partial h_z}{\partial y} - h^{-1}\frac{\partial h_y}{\partial \zeta} = \frac{4\pi\sigma}{c}\left[E_x + \frac{1}{c}\left(H_3\frac{\partial u_y}{\partial t} - H_2\frac{\partial u_z}{\partial t}\right)\right] + \frac{1}{c}\frac{\partial E_x}{\partial t}$$

$$h^{-1}\frac{\partial h_x}{\partial \zeta} - \frac{\partial h_z}{\partial x} = \frac{4\pi\sigma}{c}\left[E_y + \frac{1}{c}\left(H_1\frac{\partial u_z}{\partial t} - H_3\frac{\partial u_x}{\partial t}\right)\right] + \frac{1}{c}\frac{\partial E_y}{\partial t} \qquad (2.2)$$

$$\frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y} = \frac{4\pi\sigma}{c}\left[E_z + \frac{1}{c}\left(H_2\frac{\partial u_x}{\partial t} - H_1\frac{\partial u_y}{\partial t}\right)\right] + \frac{1}{c}\frac{\partial E_z}{\partial t}$$

$$\frac{\partial E_z}{\partial y} - h^{-1}\frac{\partial E_y}{\partial \zeta} = -\frac{1}{c}\frac{\partial h_x}{\partial t}, \qquad h^{-1}\frac{\partial E_x}{\partial \zeta} - \frac{\partial E_z}{\partial x} = -\frac{1}{c}\frac{\partial h_y}{\partial t}$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\frac{1}{c}\frac{\partial h_z}{\partial t} \tag{2.3}$$

$$\frac{\partial h_x}{\partial x} + \frac{\partial h_y}{\partial y} + h^{-1}\frac{\partial h_z}{\partial \zeta} = 0, \qquad \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + h^{-1}\frac{\partial E_z}{\partial \zeta} = 0$$

Let Q be any of the electromagnetic field and plate displacement components. Let us assign it as

$$Q = h^{-q}\sum_{s=1}^{S} h^{s-1}Q^{(s)} \tag{2.4}$$

where q is an integer, different for different magnetic field and displacement components.

Existing numerous exact solutions of plate bending problems in the absence of a magnetic field show that q must be selected in conformity with [5] for the stresses and displacements. In particular, for the displacements we shall have

$$(u_x, u_y) \rightarrow q = 2, \qquad (u_z = w) \rightarrow q = 3 \tag{2.5}$$

As regards the electromagnetic field components, various methods of selecting the number q are possible for them. Let us examine the following three versions:

$$(h_x, h_y, E_z) \rightarrow q = 1, \qquad (h_z, E_x, E_y) \rightarrow q = 2 \tag{2.6.1}$$

$$(h_x, h_y, E_z) \rightarrow q = 2, \qquad (h_z, E_x, E_y) \rightarrow q = 3 \tag{2.6.2}$$

$$(h_x, h_y, E_z) \rightarrow q = 3, \qquad (h_z, E_x, E_y) \rightarrow q = 4 \tag{2.6.3}$$

Representing the electromagnetic field and plate displacement components in (2.2) and (2.3) in the form (2.4)–(2.6), let us equate coefficients of equivalent powers of h in the left and right sides of each equation taken separately. The equations obtained from (2.3) agree in all three versions, and are the following:

$$\frac{\partial E_y^{(s)}}{\partial \zeta} = \frac{\partial E_z^{(s-2)}}{\partial y} + \frac{1}{c}\frac{\partial h_x^{(s-2)}}{\partial t}, \qquad \frac{\partial E_x^{(s)}}{\partial \zeta} = \frac{\partial E_z^{(s-2)}}{\partial x} - \frac{1}{c}\frac{\partial h_y^{(s-2)}}{\partial t}$$

$$\frac{\partial E_y^{(s)}}{\partial x} - \frac{\partial E_x^{(s)}}{\partial y} = -\frac{1}{c}\frac{\partial h_z^{(s)}}{\partial t} \tag{2.7}$$

$$\frac{\partial h_z^{(s)}}{\partial \zeta} = -\frac{\partial h_x^{(s-2)}}{\partial x} - \frac{\partial h_y^{(s-2)}}{\partial y}, \qquad \frac{\partial E_x^{(s)}}{\partial x} + \frac{\partial E_y^{(s)}}{\partial y} + \frac{\partial E_z^{(s)}}{\partial \zeta} = 0$$

Equations (2.2) result in distinct equations for each version separately. In particular:

For the first version (2.6.1)

$$\frac{\partial h_z^{(s-1)}}{\partial y} - \frac{\partial h_y^{(s-1)}}{\partial \zeta} = \frac{4\pi\sigma}{c}\left[E_x^{(s-1)} + \frac{1}{c}\left(H_3\frac{\partial u_y^{(s-1)}}{\partial t} - H_2\frac{\partial u_z^{(s)}}{\partial t}\right)\right] + \frac{1}{c}\frac{\partial E_x^{(s-1)}}{\partial t}$$

$$\frac{\partial h_x^{(s-1)}}{\partial \zeta} - \frac{\partial h_z^{(s-1)}}{\partial x} = \frac{4\pi\sigma}{c}\left[E_y^{(s-1)} + \frac{1}{c}\left(H_1\frac{\partial u_z^{(s)}}{\partial t} - H_3\frac{\partial u_x^{(s-1)}}{\partial t}\right)\right] + \frac{1}{c}\frac{\partial E_y^{(s-1)}}{\partial t} \tag{2.8}$$

$$\frac{\partial h_y^{(s-1)}}{\partial x} - \frac{\partial h_x^{(s-1)}}{\partial y} = \frac{4\pi\sigma}{c}\left[E_z^{(s-1)} + \frac{1}{c}\left(H_2\frac{\partial u_x^{(s)}}{\partial t} - H_1\frac{\partial u_y^{(s)}}{\partial t}\right)\right] + \frac{1}{c}\frac{\partial E_z^{(s-1)}}{\partial t}$$

For the second version (2.6.2)

$$\begin{aligned}
\frac{\partial h_z^{(s)}}{\partial y}-\frac{\partial h_y^{(s)}}{\partial \zeta}&=\frac{4\pi\sigma}{c}\left[E_x^{(s)}+\frac{1}{c}\left(H_3\frac{\partial u_y^{(s-1)}}{\partial t}-H_2\frac{\partial u_z^{(s-1)}}{\partial t}\right)\right]+\frac{1}{c}\frac{\partial E_x^{(s)}}{\partial t}\\
\frac{\partial h_x^{(s)}}{\partial \zeta}-\frac{\partial h_z^{(s)}}{\partial x}&=\frac{4\pi\sigma}{c}\left[E_y^{(s)}+\frac{1}{c}\left(H_1\frac{\partial u_z^{(s)}}{\partial t}-H_3\frac{\partial u_x^{(s-1)}}{\partial t}\right)\right]+\frac{1}{c}\frac{\partial E_y^{(s)}}{\partial t} \qquad (2.9)\\
\frac{\partial h_y^{(s)}}{\partial x}-\frac{\partial h_x^{(s)}}{\partial y}&=\frac{4\pi\sigma}{c}\left[E_z^{(s)}+\frac{1}{c}\left(H_2\frac{\partial u_x^{(s)}}{\partial t}-H_1\frac{\partial u_y^{(s)}}{\partial t}\right)\right]+\frac{1}{c}\frac{\partial E_z^{(s)}}{\partial t}
\end{aligned}$$

For the third version (2. 6. 3)

$$\begin{aligned}
\frac{\partial h_z^{(s)}}{\partial y}-\frac{\partial h_y^{(s)}}{\partial \zeta}&=\frac{4\pi\sigma}{c}\left[E_x^{(s)}+\frac{1}{c}\left(H_3\frac{\partial u_y^{(s-2)}}{\partial t}-H_2\frac{\partial u_z^{(s-1)}}{\partial t}\right)\right]+\frac{1}{c}\frac{\partial E_x^{(s)}}{\partial t}\\
\frac{\partial h_x^{(s)}}{\partial \zeta}-\frac{\partial h_z^{(s)}}{\partial x}&=\frac{4\pi\sigma}{c}\left[E_y^{(s)}+\frac{1}{c}\left(H_1\frac{\partial u_z^{(s-1)}}{\partial t}-H_3\frac{\partial u_x^{(s-2)}}{\partial t}\right)\right]+\frac{1}{c}\frac{\partial E_y^{(s)}}{\partial t} \qquad (2.10)\\
\frac{\partial h_y^{(s)}}{\partial x}-\frac{\partial h_x^{(s)}}{\partial y}&=\frac{4\pi\sigma}{c}\left[E_z^{(s)}+\frac{1}{c}\left(H_2\frac{\partial u_x^{(s-1)}}{\partial t}-H_1\frac{\partial u_y^{(s-1)}}{\partial t}\right)\right]+\frac{1}{c}\frac{\partial E_z^{(s)}}{\partial t}
\end{aligned}$$

Equations (2. 7) and (2. 8) in the first version, (2. 7) and (2. 9) in the second version, and (2. 7) and (2. 10) in the third version form a chain of systems of equations of the fundamental iteration process in the successive ($s = 1, 2, 3, \ldots$) determination of the desired $Q^{(s)}$, for each version separately. It must here be considered that $Q^{(s)} \equiv 0$ for $s < 1$ and also that the appropriate quantites $Q^{(1)}, Q^{(2)}, \ldots, Q^{(s)}$ are considered known in constructing $Q^{(s+1)}$.

Obtained in a first equation from (2. 8) for the first version is

$$\frac{4\pi\sigma}{c^2}H_2\frac{\partial u_z^{(1)}}{\partial t}=0, \qquad \frac{4\pi\sigma}{c^2}H_1\frac{\partial u_z^{(1)}}{\partial t}=0, \quad \frac{4\pi\sigma}{c^2}\left(H_2\frac{\partial u_x^{(1)}}{\partial t}-H_1\frac{\partial u_y^{(1)}}{\partial t}\right)=0$$

For the nonstationary problem this can only be when

$$H_1 = H_2 = 0$$

Therefore, the first version is possible when the given external magnetic field is perpendicular to the plane of the plate [$\mathbf{H}_0 = \mathbf{H}_0\ (0, 0, H_3)$]. Assuming $H_1 = H_2 = 0$, we obtain the following equations from (2. 2) in place of (2. 8):

$$\begin{aligned}
\frac{\partial h_z^{(s)}}{\partial y}-\frac{\partial h_y^{(s)}}{\partial \zeta}&=\frac{4\pi\sigma}{c}\left[E_x^{(s)}+\frac{1}{c}H_3\frac{\partial u_y^{(s)}}{\partial t}\right]+\frac{1}{c}\frac{\partial E_x^{(s)}}{\partial t}\\
\frac{\partial h_x^{(s)}}{\partial \zeta}-\frac{\partial h_z^{(s)}}{\partial x}&=\frac{4\pi\sigma}{c}\left[E_y^{(s)}-\frac{1}{c}H_3\frac{\partial u_x^{(s)}}{\partial t}\right]+\frac{1}{c}\frac{\partial E_y^{(s)}}{\partial t} \qquad (2.11)\\
\frac{\partial h_y^{(s)}}{\partial x}-\frac{\partial h_x^{(s)}}{\partial y}&=\frac{4\pi\sigma}{c}E_z^{(s)}+\frac{1}{c}\frac{\partial E_z^{(s)}}{\partial t}
\end{aligned}$$

In the second version there is no constraint on the given external magnetic field, and even in the first approximation the electrodynamics equations (2. 7) and (2. 9) are not separated from the equations of elastic plate vibrations and they must be considered jointly.

As is seen from (2. 7) and (2. 10), in the case of the third version the electromagnetic field components are determined in a first approximation independently of the elastic displacements. It should therefore be assumed that the third version is acceptable in those magnetoelasticity problems for which it is known that the elastic vibrations influence the change in the electromagnetic field slightly.

There results from the above that the second version is most general.

The solution of the system (2.7) and (2.11), (2.7) and (2.9), (2.7) and (2.10) are represented in the form of two components $Q^{(s)} = Q_i^{(s)} + Q^{*(s)}$. The first component is interpreted as the integral of the homogeneous system obtained by discarding quantities with superscript less than s, and the second member is understood to be some particular integral of the mentioned system in which all quantities with superscript less than s, are considered known.

Those equations of the homogeneous system which are obtained from (2.7) are common to all three versions and have the form:

$$\frac{\partial E_x^{(s)}}{\partial \zeta} = 0, \qquad \frac{\partial E_y^{(s)}}{\partial \zeta} = 0, \qquad \frac{\partial h_z^{(s)}}{\partial \zeta} = 0 \tag{2.12}$$

$$\frac{\partial E_y^{(s)}}{\partial x} - \frac{\partial E_x^{(s)}}{\partial y} = -\frac{1}{c}\frac{\partial h_z^{(s)}}{\partial t}, \qquad \frac{\partial E_x^{(s)}}{\partial x} + \frac{\partial E_y^{(s)}}{\partial y} + \frac{\partial E_z^{(s)}}{\partial \zeta} = 0$$

The remaining equations of the homogeneous system are:

a) For the first version according to (2.11)

$$\frac{\partial h_z^{(s)}}{\partial y} - \frac{\partial h_y^{(s)}}{\partial \zeta} = \frac{4\pi\sigma}{c}\left[E_x^{(s)} + \frac{1}{c}H_3\frac{\partial u_y^{(s)}}{\partial t}\right] + \frac{1}{c}\frac{\partial E_x^{(s)}}{\partial t}$$

$$\frac{\partial h_x^{(s)}}{\partial \zeta} - \frac{\partial h_z^{(s)}}{\partial x} = \frac{4\pi\sigma}{c}\left[E_y^{(s)} - \frac{1}{c}H_3\frac{\partial u_x^{(s)}}{\partial t}\right] + \frac{1}{c}\frac{\partial E_y^{(s)}}{\partial t}$$

$$\frac{\partial h_y^{(s)}}{\partial x} - \frac{\partial h_x^{(s)}}{\partial y} = \frac{4\pi\sigma}{c}E_z^{(s)} + \frac{1}{c}\frac{\partial E_z^{(s)}}{\partial t} \tag{2.13}$$

b) For the second version according to (2.9)

$$\frac{\partial h_z^{(s)}}{\partial y} - \frac{\partial h_y^{(s)}}{\partial \zeta} = \frac{4\pi\sigma}{c}\left[E_x^{(s)} - \frac{H_2}{c}\frac{\partial u_z^{(s)}}{\partial t}\right] + \frac{1}{c}\frac{\partial E_x^{(s)}}{\partial t}$$

$$\frac{\partial h_x^{(s)}}{\partial \zeta} - \frac{\partial h_z^{(s)}}{\partial x} = \frac{4\pi\sigma}{c}\left[E_y^{(s)} + \frac{H_1}{c}\frac{\partial u_z^{(s)}}{\partial t}\right] + \frac{1}{c}\frac{\partial E_y^{(s)}}{\partial t} \tag{2.14}$$

$$\frac{\partial h_y^{(s)}}{\partial x} - \frac{\partial h_x^{(s)}}{\partial y} = \frac{4\pi\sigma}{c}\left[E_z^{(s)} + \frac{H_2}{c}\frac{\partial u_x^{(s)}}{\partial t} - \frac{H_1}{c}\frac{\partial u_y^{(s)}}{\partial t}\right] + \frac{1}{c}\frac{\partial E_z^{(s)}}{\partial t}$$

c) For the third version according to (2.10)

$$\frac{\partial h_z^{(s)}}{\partial y} - \frac{\partial h_y^{(s)}}{\partial \zeta} = \frac{4\pi\sigma}{c}E_x^{(s)} + \frac{1}{c}\frac{\partial E_x^{(s)}}{\partial t}$$

$$\frac{\partial h_x^{(s)}}{\partial \zeta} - \frac{\partial h_z^{(s)}}{\partial x} = \frac{4\pi\sigma}{c}E_y^{(s)} + \frac{1}{c}\frac{\partial E_y^{(s)}}{\partial t} \tag{2.15}$$

$$\frac{\partial h_y^{(s)}}{\partial x} - \frac{\partial h_x^{(s)}}{\partial y} = \frac{4\pi\sigma}{c}E_z^{(s)} + \frac{1}{c}\frac{\partial E_z^{(s)}}{\partial t}$$

The systems of equations obtained for each version separately: (2.12) and (2.13) (first version), (2.12) and (2.14) (second version), (2.12) and (2.15) (third version), are easily integrated. We hence note that the expressions $E_{xi}^{(s)}$, $E_{yi}^{(s)}$, $E_{zi}^{(s)}$, $h_{zi}^{(s)}$ in all three versions are of the identical form

$$E_{xi}^{(s)} = E_{x0}^{(s)}(x, y, t), \qquad E_{yi}^{(s)} = E_{y0}^{(s)}(x, y, t)$$

$$h_{zi}^{(s)} = h_{z0}^{(s)}(x, y, t), \qquad E_{zi}^{(s)} = -\zeta\left[\frac{\partial E_{x0}^{(s)}}{\partial x} + \frac{\partial E_{y0}^{(s)}}{\partial y}\right] \tag{2.16}$$

To determine $h_{xi}^{(s)}$ and $h_{yi}^{(s)}$ we use the relationships

$$u_{zi}^{(s)} = w_i^{(s)} = w_0^{(s)}(x, y, t), \qquad u_{xi}^{(s)} = -\zeta\frac{\partial w_0^{(s)}}{\partial x}, \qquad u_{yi}^{(s)} = -\zeta\frac{\partial w_0^{(s)}}{\partial y}$$

which have been obtained in [5] and hold independently of the presence of the magnetic field. We then obtain from (2.13)–(2.15)

For the first version

$$h_{xi}^{(s)} = \zeta\left[\frac{\partial h_{z0}^{(s)}}{\partial x} + \frac{4\pi\sigma}{c}\left(E_{y0}^{(s)} + \zeta\frac{H_3}{2c}\frac{\partial^2 w_0^{(s)}}{\partial t\partial x}\right) + \frac{1}{c}\frac{\partial E_{y0}^{(s)}}{\partial t}\right]$$

$$h_{yi}^{(s)} = \zeta\left[\frac{\partial h_{z0}^{(s)}}{\partial y} - \frac{4\pi\sigma}{c}\left(E_{x0}^{(s)} - \zeta\frac{H_3}{2c}\frac{\partial^2 w_0^{(s)}}{\partial t\partial y}\right) - \frac{1}{c}\frac{\partial E_{x0}^{(s)}}{\partial t}\right]$$

For the second version

$$h_{xi}^{(s)} = \zeta\left[\frac{\partial h_{z0}^{(s)}}{\partial x} + \frac{4\pi\sigma}{c}\left(E_{y0}^{(s)} + \frac{H_1}{c}\frac{\partial w_0^{(s)}}{\partial t}\right) + \frac{1}{c}\frac{\partial E_{y0}^{(s)}}{\partial t}\right]$$

$$h_{yi}^{(s)} = \zeta\left[\frac{\partial h_{z0}^{(s)}}{\partial y} - \frac{4\pi\sigma}{b}\left(E_{x0}^{(s)} - \frac{H_2}{c}\frac{\partial w_0^{(s)}}{\partial t}\right) - \frac{1}{c}\frac{\partial E_{x0}^{(s)}}{\partial t}\right]$$

For the third version

$$h_{xi}^{(s)} = \zeta\left[\frac{\partial h_{z0}^{(s)}}{\partial x} + \frac{4\pi\sigma}{c}E_{y0}^{(s)} + \frac{1}{c}\frac{\partial E_{y0}^{(s)}}{\partial t}\right]$$

$$h_{yi}^{(s)} = \zeta\left[\frac{\partial h_{z0}^{(s)}}{\partial y} - \frac{4\pi\sigma}{c}E_{x0}^{(s)} - \frac{1}{c}\frac{\partial E_{x0}^{(s)}}{\partial t}\right]$$

Let us find the integrals of the systems of equations (2.7) and (2.11), (2.7) and (2.9), (2.7) and (2.10). For all three versions we obtain from (2.7)

$$E_x^{*(s)} = \int_0^\zeta\left(\frac{\partial E_z^{(s-2)}}{\partial x} - \frac{1}{c}\frac{\partial h_y^{(s-2)}}{\partial t}\right)d\zeta, \qquad E_z^{*(s)} = -\int_0^\zeta\left(\frac{\partial E_x^{*(s)}}{\partial x} + \frac{\partial E_y^{*(s)}}{\partial y}\right)d\zeta \tag{2.17}$$

$$E_y^{*(s)} = \int_0^\zeta\left(\frac{\partial E_z^{(s-2)}}{\partial y} + \frac{1}{c}\frac{\partial h_x^{(s-2)}}{\partial t}\right)d\zeta, \qquad h_z^{*(s)} = -\int_0^\zeta\left(\frac{\partial h_x^{(s-2)}}{\partial x} + \frac{\partial h_y^{(s-2)}}{\partial y}\right)d\zeta$$

The expressions for $h_x^{*(s)}$ and $h_y^{*(s)}$ are found from (2.11), (2.9) and (2.10) as follows:

For the first version

$$h_x^{*(s)} = \int_0^\zeta\left[\frac{\partial h_z^{*(s)}}{\partial x} + \frac{4\pi\sigma}{c}\left(E_y^{*(s)} - \frac{H_3}{c}\frac{\partial u_x^{*(s)}}{\partial t}\right) + \frac{1}{c}\frac{\partial E_y^{*(s)}}{\partial t}\right]d\zeta$$

$$h_y^{*(s)} = \int_0^\zeta\left[\frac{\partial h_z^{*(s)}}{\partial y} - \frac{4\pi\sigma}{c}\left(E_x^{*(s)} + \frac{H_3}{c}\frac{\partial u_y^{*(s)}}{\partial t}\right) + \frac{1}{c}\frac{\partial E_x^{*(s)}}{\partial t}\right]d\zeta$$

For the second version

$$h_x^{*(s)}=\int_0^\zeta\left\{\frac{\partial h_z^{*(s)}}{\partial x}+\frac{4\pi\sigma}{c}\left[E_y^{*(s)}+\frac{1}{c}\left(H_1\frac{\partial w^{*(s)}}{\partial t}-H_3\frac{\partial u_y^{(s-1)}}{\partial t}\right]\right)+\frac{1}{c}\frac{\partial E_y^{*(s)}}{\partial t}\right\}d\zeta$$

$$h_y^{*(s)}=\int_0^\zeta\left\{\frac{\partial h_z^{*(s)}}{\partial y}-\frac{4\pi\sigma}{c}\left[E_x^{*(s)}+\frac{1}{c}\left(H_3\frac{\partial u_y^{(s-1)}}{\partial t}-H_2\frac{\partial w^{*(s)}}{\partial t}\right)\right]-\frac{1}{c}\frac{\partial E_x^{*(s)}}{\partial t}\right\}d\zeta$$

For the third version

$$h_x^{*(s)}=\int_0^\zeta\left\{\frac{\partial h_z^{*(s)}}{\partial x}+\frac{4\pi\sigma}{c}\left[E_y^{*(s)}+\frac{1}{c}\left(H_1\frac{\partial w^{(s-1)}}{\partial t}-H_3\frac{\partial u_x^{(s-2)}}{\partial t}\right)\right]+\frac{1}{c}\frac{\partial E_y^{*(s)}}{\partial t}\right\}d\zeta$$

$$h_y^{*(s)}=\int_0^\zeta\left\{\frac{\partial h_z^{*(s)}}{\partial y}-\frac{4\pi\sigma}{c}\left[E_x^{*(s)}+\frac{1}{c}\left(H_3\frac{\partial u_y^{(s-2)}}{\partial t}-H_2\frac{\partial w^{(s-1)}}{\partial t}\right)\right]+\frac{1}{c}\frac{\partial E_x^{*(s)}}{\partial t}\right\}d\zeta$$

All the quantities with an asterisk here are functions of the variables (x, y, ζ, t) and the quantities without the asterisk and with superscript less than s are considered known.

As has been mentioned earlier, $Q^{(s)}\equiv 0$ for $s<1$. Hence, from (2.17) as well as [5], from which information on u_x, u_y, w is taken, it follows that $Q^{*(1)}$ and $Q^{*(2)}$ which are marked with an asterisk, are identically zero for $s=1$ and $s=2$.

Discussions on the asymptotic integration of systems of equations, which are expounded in detail in [5], have been repeated in this Section for the case of (1.4) and (1.5). Further discussions on the asymptotic integration of systems of equations are omitted because what has been presented is adequate for the asymptotic method of integrating the three-dimensional equations of magnetoelasticity to be treated as the method of formulating well-founded hypotheses and for the formulation of initial hypotheses for the problem posed.

In a first approximation the fundamental iteration process applied to the equations of elasticity theory reduce to the theory of plate bending underlying which is the hypothesis of undeformable normals [5, 6].

Examining the solutions (2.16) and (2.17) obtained above for the linearized equations (1.4), we note that the quantities E_x, E_y and h_z are independent of the coordinate ζ to the third approximation of the asymptotic integration.

On the basis of the above the following hypotheses can be formulated for the interior problem of magnetoelasticity.

a) After deformation a straight line element of the plate normal to the middle plane remains a straight line normal to the deformed middle surface of the plate and retains its length.

b) Tangential components of the excited electrical field intensity and the normal component of the excited magnetic field intensity remain invariant over the plate thickness.

To the accuracy of the first assumption it should also be assumed that the nomal stresses σ_z can be neglected as compared with the other stresses. The second hypothesis can be considered as some electrodynamic analog of the hypothesis of undeformable normals.

3. The hypotheses formulated above for the interior problem are written analytically

as follows:

$$u_x = -z\frac{\partial w}{\partial x}, \quad u_y = -z\frac{\partial w}{\partial y}, \quad u_z = w(x,y,t) \tag{3.1}$$

$$E_x = \varphi(x,y,t), \quad E_y = \psi(x,y,t), \quad h_z = f(x,y,t)$$

here w is the desired normal displacement of the plate, φ, ψ, f are the desired components of the corresponding electromagnetic field intensities.

Assuming the hypotheses (a) and (b), i. e. (3.1), the interior three-dimensional magnetoelasticity problem reduces substantially to a two-dimensional problem of magnetoelasticity of a plate.

According to (3.1) we obtain from the linearized equations (1.4)

$$\frac{\partial h_x}{\partial z} = \frac{\partial f}{\partial x} + \frac{4\pi\sigma}{c}\left[\psi + \frac{1}{c}\left(H_1\frac{\partial w}{\partial t} + zH_3\frac{\partial^2 w}{\partial x\partial t}\right)\right] + \frac{1}{c}\frac{\partial \psi}{\partial t}$$

$$\frac{\partial h_y}{\partial z} = \frac{\partial f}{\partial y} - \frac{4\pi\sigma}{c}\left[\varphi - \frac{1}{c}\left(H_2\frac{\partial w}{\partial t} + zH_3\frac{\partial^2 w}{\partial y\partial t}\right)\right] - \frac{1}{c}\frac{\partial \varphi}{\partial t}$$

$$\frac{\partial E_z}{\partial z} = -\frac{\partial \varphi}{\partial x} - \frac{\partial \psi}{\partial y} \tag{3.2}$$

Integrating the representations (3.2) with respect to z between zero and z, and taking into account the conditions of electromagnetic field continuity for the whole space under consideration

$$h_x = h_x^+, \quad h_y = h_y^+, \quad E_z = E_z^+ \quad \text{for } z = h$$

$$h_x = h_x^-, \quad h_y = h_y^-, \quad E_z = E_z^- \quad \text{for } z = -h \tag{3.3}$$

we obtain for the remaining electromagnetic field components

$$h_x = \frac{h_x^+ + h_x^-}{2} + z\left[\frac{\partial f}{\partial x} + \frac{1}{c}\frac{\partial \psi}{\partial t} + \frac{4\pi\sigma}{c}\left(\psi + \frac{H_1}{c}\frac{\partial w}{\partial t}\right)\right] + \frac{4\pi\sigma H_3}{c^2}\frac{z^2 - h^2}{2}\frac{\partial^2 w}{\partial x\partial t}$$

$$h_y = \frac{h_y^+ + h_y^-}{2} + z\left[\frac{\partial f}{\partial y} - \frac{1}{c}\frac{\partial \varphi}{\partial t} - \frac{4\pi\sigma}{c}\left(\varphi - \frac{H_2}{c}\frac{\partial w}{\partial t}\right)\right] + \frac{4\pi\sigma H_3}{c^2}\frac{z^2 - h^2}{2}\frac{\partial^2 w}{\partial y\partial t}$$

$$E_z = \frac{E_z^+ + E_z^-}{2} - z\left(\frac{\partial \varphi}{\partial x} + \frac{\partial \psi}{\partial y}\right) \tag{3.4}$$

Therefore, all the electromagnetic field components are represented by means of the four desired functions φ, ψ, f, w by using (3.1) and (3.4).

According to the hypothesis of undeformable normals, there are the following relationships [5, 6] together with (3.1):

$$\sigma_x = -z\frac{E}{1-\nu^2}\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right) \tag{3.5}$$

$$\sigma_y = -z\frac{E}{1-\nu^2}\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right), \qquad \tau_{xy} = -z\frac{E}{1+\nu}\frac{\partial^2 w}{\partial x\partial y}$$

where E and ν are, respectively, the elastic modulus and Poisson's ratio of the plate material.

By virtue of (3.1) and (3.5) and taking account of conditions on the plate surfaces

$$\tau_{xz}=\tau_{yz}=0, \qquad \sigma_z = p(x, y, t) \quad \text{при } z=h$$
$$\tau_{xz}=\tau_{yz}=\sigma_z=0 \quad \text{при } z=-h \tag{3.6}$$

we obtain for the transverse tangential stresses in the plate from the first two equations of (1.5)

$$\tau_{xz} = -z\frac{\sigma}{c}\left(H_3\psi - H_2\frac{E_z^+ + E_z^-}{2} + \frac{H_1H_2}{c}\frac{\partial w}{\partial t}\right) + $$
$$+\frac{h^2-z^2}{2}\left\{\rho\frac{\partial^3 w}{\partial x\,\partial t^2} - \frac{E}{1-\nu^2}\frac{\partial}{\partial x}\Delta w + \frac{\sigma}{c}\left[H_2\left(\frac{\partial\varphi}{\partial x}+\frac{\partial\psi}{\partial y}\right) + \right.\right.$$
$$\left.\left. + (H_3^2 + H_2^2)\frac{1}{c}\frac{\partial^2 w}{\partial x\,\partial t} - \frac{H_1H_2}{c}\frac{\partial^2 w}{\partial y\,\partial t}\right]\right\} \tag{3.7}$$
$$\tau_{yz} = -z\frac{\sigma}{c}\left(H_1\frac{E_z^+ + E_z^-}{2} - H_3\varphi + \frac{H_2H_3}{c}\frac{\partial w}{\partial t}\right) +$$
$$+\frac{h^2-z^2}{2}\left\{\rho\frac{\partial^3 w}{\partial y\,\partial t^2} - \frac{E}{1-\nu^2}\frac{\partial}{\partial y}\Delta w + \frac{\sigma}{c}\left[(H_1^2+H_3^2)\frac{1}{c}\frac{\partial^2 w}{\partial y\,\partial t} - \right.\right.$$
$$\left.\left. - H_1\left(\frac{\partial\varphi}{\partial x}+\frac{\partial\psi}{\partial y}\right) - \frac{H_1H_2}{c}\frac{\partial^2 w}{\partial x\,\partial t}\right]\right\} \quad \left(\Delta = \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)$$

Substituting (3.1) and (3.4) into (1.4), and (3.1) and (3.7) into the third equation of (1.5) and integrating the equations hence obtained with respect to z between $z=-h$ and $z=h$ taking (3.6) into account, we obtain the following complete system of differential equations in the desired functions φ, ψ, f and w:

$$\frac{\partial\psi}{\partial x} - \frac{\partial\varphi}{\partial y} = -\frac{1}{c}\frac{\partial f}{\partial t}, \quad \frac{\partial f}{\partial x} + \frac{4\pi\sigma}{c}\left(\psi + \frac{H_1}{c}\frac{\partial w}{\partial t}\right) + \frac{1}{c}\frac{\partial\psi}{\partial t} = \frac{h_x^+ - h_x^-}{2h}$$
$$\frac{\partial f}{\partial y} - \frac{4\pi\sigma}{c}\left(\varphi - \frac{H_2}{c}\frac{\partial w}{\partial t}\right) - \frac{1}{c}\frac{\partial\varphi}{\partial t} = \frac{h_y^+ - h_y^-}{2h}$$
$$D\Delta\Delta w - \frac{2}{3}\rho h^3\frac{\partial}{\partial t}\Delta w - \frac{2\sigma h^3}{3c}\left(H_2\frac{\partial^2\varphi}{\partial x^2} - H_1\frac{\partial^2\varphi}{\partial x\,\partial y} + H_2\frac{\partial^2\psi}{\partial x\,\partial y} - H_1\frac{\partial^2\psi}{\partial y^2}\right) -$$
$$-\frac{2\sigma h^3}{3c^2}\frac{\partial}{\partial t}\left[(H_2^2+H_3^2)\frac{\partial^2 w}{\partial x^2} + (H_1^2+H_3^2)\frac{\partial^2 w}{\partial y^2} - 2H_1H_2\frac{\partial^2 w}{\partial x\,\partial y}\right] -$$
$$-2\frac{\sigma h}{c}\left[H_2\left(\varphi - \frac{H_2}{c}\frac{\partial w}{\partial t}\right) - H_1\left(\psi + \frac{H_1}{c}\frac{\partial w}{\partial t}\right)\right] + 2\rho h\frac{\partial^2 w}{\partial t^2} = p(x, y, t)$$
$$\left(D = \frac{2Eh^3}{3(1-\nu^2)}\right) \tag{3.8}$$

The following conditions, which connect the boundary values of the electromagnetic field intensity components, are also obtained from the systems (1.4) upon execution of the above-mentioned operation:

$$\frac{1}{2}\frac{\partial}{\partial x}(h_x^+ + h_x^-) + \frac{1}{2}\frac{\partial}{\partial y}(h_y^+ + h_y^-) - \frac{4\pi\sigma H_3 h^2}{3c^2}\frac{\partial}{\partial t}\Delta w = 0$$
$$\frac{1}{2}\frac{\partial}{\partial x}(h_y^+ + h_y^-) - \frac{1}{2}\frac{\partial}{\partial y}(h_x^+ + h_x^-) = \frac{2\pi\sigma}{c}(E_z^+ + E_z^-) + \frac{1}{2c}\frac{\partial}{\partial t}(E_z^+ + E_z^-)$$
$$\frac{1}{2}\frac{\partial}{\partial y}(E_z^+ + {}_z^-) + -\frac{1}{2c}\frac{\partial}{\partial t}(h_x^+ + h_x^-) + \frac{4\pi\sigma H_3 h^2}{3c^3}\frac{\partial^3 w}{\partial x\,\partial t^2} \tag{3.9}$$
$$\frac{1}{2}\frac{\partial}{\partial x}(E_z^+ + E_z^-) = \frac{1}{2c}\frac{\partial}{\partial t}(h_y^+ + h_y^-) - \frac{4\pi\sigma H_3 h^2}{3c^3}\frac{\partial^3 w}{\partial y\,\partial t^2}$$
$$\frac{\partial\varphi}{\partial x} + \frac{\partial\psi}{\partial y} = -\frac{E_z^+ - E_z^-}{2h}$$

and from the system (1.5) the conditions

$$H_3\left(\psi+\frac{H_1}{c}\frac{\partial w}{\partial t}\right)-H_2\frac{E_z^{+}+E_z^{-}}{2}=0,\quad H_3\left(\varphi-\frac{H_2}{c}\frac{\partial w}{\partial t}\right)-H_1\frac{E_z^{+}+E_z^{-}}{2}=0 \tag{3.10}$$

Therefore, the problem of the vibration of an electrically conducting isotropic plate in an external magnetic field, formulated in Sect. 1, leads to the joint solution of the system of differential equations (3.8) of magnetoelasticity for the interior domain of the plate, and the electrodynamics equations (1.3) in the exterior domain. Conditions (3.9) and $\operatorname{div} \mathbf{E}^{(n)}=0$, $\operatorname{div}\mathbf{H}^{(n)}=0$ ($n=1$ for $z\geqslant h$ and $n=2$ for $z\leqslant -h$) will be the boundary conditions of the problem in addition to the customary plate fixing conditions and the continuity conditions for the quantities E_x, E_y, h_z on the plate surface.

4. As an illustration, let us apply the method expounded above to solve the problem of vibrations of an infinite plate with constant finite electrical conductivity in the presence of an external magnetic field with intensity vector parallel to the x-axis. For simplicity, cylindrical bending is considered, and rotational inertia and moments $\mathbf{R}$ (X, Y, Z) are neglected.

It follows from the conditions of the problem that $E_x=E_z=h_y=0$ in the whole space. Conditions (3.10) are satisfied identically. In this case, the system of equations (3.8) takes the form

$$\frac{\partial\psi}{\partial x}+\frac{1}{c}\frac{\partial f}{\partial t}=0,\quad \frac{\partial f}{\partial x}+\frac{4\pi\sigma}{c}\left(\psi+\frac{H_1}{c}\frac{\partial w}{\partial t}\right)+\frac{1}{c}\frac{\partial\psi}{\partial t}=\frac{h_x^{+}-h_x^{-}}{2h}$$

$$D\frac{\partial^4 w}{\partial x^4}+\frac{2\sigma hH_1}{c}\left(\psi+\frac{H_1}{c}\frac{\partial w}{\partial t}\right)+2\rho h\frac{\partial^2 w}{\partial t^2}=0 \tag{4.1}$$

The electrodynamics equations in the exterior domain (1.3) are converted to the form

$$\Delta h_x^{(n)}-\frac{1}{c^2}\frac{\partial^2 h_x^{(n)}}{\partial t^2}=0,\qquad \Delta h_z^{(n)}-\frac{1}{c^2}\frac{\partial^2 h_z^{(n)}}{\partial t^2}=0 \tag{4.2}$$

$$\Delta E_y^{(n)}-\frac{1}{c^2}\frac{\partial^2 E_y^{(n)}}{\partial t^2}=0$$

while the boundary conditions of the problem will be

$$\frac{\partial}{\partial x}(h_x^{+}+h_x^{-})=\frac{\partial}{\partial t}(h_x^{+}+h_x^{-})=0,\qquad \frac{\partial h_x^{(n)}}{\partial x}+\frac{\partial h_z^{(n)}}{\partial z}=0$$

$$E_y^{(1)}\big|_{z=h}=E_y^{(2)}\big|_{z=-h}=\psi(x,t),\qquad h_z^{(1)}\big|_{z=h}=h_z^{(2)}\big|_{z=-h}=f(x,t) \tag{4.3}$$

and the boundedness condition for the perturbations at infinity.

We seek the solution of (4.1) and (4.2) in the form of waves being propagated along the x-axis

$$w=w_0e^{i(\omega t-kx)},\qquad \psi=\psi_0e^{i(\omega t-kx)}$$

$$f=f_0e^{i(\omega t-kx)},\qquad E_y^{(n)}=\Psi_n(z)\,e^{i(\omega t-kx)} \tag{4.4}$$

$$h_x^{(n)}=\Phi_n(z)\,e^{i(\omega t-kx)},\qquad h_z^{(n)}=F_n(z)\,e^{i(\omega t-kx)}$$

Here, all the functions of z are unknown and to be determined, $k=\pi/\lambda$ is the wave number, λ is the half-wavelength, ω the vibration frequency. Substituting (4.4) into (4.2) we obtain the equations defining the mentioned unknown functions

$$\Phi_n'' - \left(k^2 - \frac{\omega^2}{c^2}\right)\Phi_n = 0, \; F_n'' - \left(k^2 - \frac{\omega^2}{c^2}\right) F_n = 0, \; \Psi_n'' - \left(k^2 - \frac{\omega^2}{c^2}\right)\Psi_n = 0$$

Therefore

$$\Phi_1 = A_1 e^{-k_1 z}, \quad \Phi_2 = A_2 e^{k_1 z}, \quad F_1 = B_1 e^{-k_1 z}, \quad F_2 = B_2 e^{k_1 z}$$

$$\Psi_1 = C_1 e^{-k_1 z}, \quad \Psi_2 = C_2 e^{k_1 z}, \quad k_1 = \sqrt{k^2 - \frac{\omega^2}{c^2}} \tag{4.5}$$

Satisfying the first two equations of the system (4.1) and the boundary conditions (4.3), we find the magnitudes of the induced magnetic and electric field intensity components in the whole space as a function of the plate deflections

$$h_x = z\frac{qk_1}{h} w, \quad h_z = -ikqw, \quad E_y = -\frac{i\omega q}{c} w$$

$$h_x^{(1)} = qk_1 e^{k_1(h-z)} w, \qquad h_x^{(2)} = -qk_1 e^{k_1(h+z)} w \tag{4.6}$$

$$h_z^{(1)} = -ikqe^{k_1(h-z)} w, \qquad h_z^{(2)} = -ikqe^{k_1(h+z)} w$$

$$q = H_1 \left[1 + \frac{c^2 k_1 (1 + k_1 h)}{i\omega 4\pi\sigma h}\right]^{-1}$$

Substituting (4.4) and (4.6) into the third equations of the system (4.1), we obtain an equation to determine the vibrations frequency

$$\omega^2 - \Omega^2 - \frac{H_1 k_1 (1 + k_1 h)}{4\pi\rho h} q = 0 \qquad \left(\Omega^2 = \frac{Dk^4}{2\rho h}\right) \tag{4.7}$$

where Ω is the natural plate vibrations frequency in the absence of a magnetic field.

Comparing the values of the quantities in (4.6) with the corresponding values of the same quantities obtained in [2] on the basis of the exact solution, shows that the results found by solving the problem by the method proposed agree with the first approximation of the results of the exact solution expanded in a power series in νz. In conformity with [2]

$$\nu = k_1^2 + \frac{1}{c^2} i\omega 4\pi\sigma$$

The first approximation of the exact solution is obtained from the mentioned expansion by neglecting the quantities $|\nu^2| h^2$ as compared with unity.

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